Homework # 1: 2.2, 2.3, 2.8, 2.17, 2.19

2.2) *a.* What is the volume of air required to contain 1 kg of CO_2?

Solution:

In order to solve this problem, we will first determine what weight of CO_2 is needed; than we will figure out how many moles of atmosphere molecules we need, and finally determine the volume of that many moles.

$1\ kg\ C \times \frac{44\ g\ CO_2}{12\ g\ C} \times \frac{1000\ g}{kg} = 3.67 \times 10^3 g\ CO_2$ needed.

weight of 1 mole of atmosphere [1] $\approx .20 \times 32\ \frac{g}{mol} + .80 \times 28\ \frac{g}{mol} = 28.8$ g atmosphere / mol.
weight of CO_2 in 1 mole of atmosphere $= \frac{28.8\ g\ atmosphere}{mol} \times \frac{0.00046\ g\ CO_2}{1\ g\ atmosphere} = 0.013248$ g $\frac{CO_2}{mol}$
\# of moles needed $= 3.67 \times 10^3\ g\ CO_2 \times \frac{mol}{0.013248\ g\ CO_2} = 2.77 \times 10^5\ moles$

Finally, the volume can be found using the ideal gas law:
$V = \frac{nRT}{p} = \frac{(2.77 \times 10^5\ mol)(.08206 L \cdot atm/K)(298K)}{1\ atm} = 6.77 \times 10^6\ L$

b. Determine the mass of carbon in 1 square meter of air.

Solution

This can be solved pretty quickly using dimensional analysis:
$\frac{1.033 \times 10^4\ kg}{m^2} \times 1\ m^2 \times \frac{10^3\ g}{kg} \times \frac{0.0046\ g\ CO_2}{g\ atmosphere} \times \frac{12\ g\ C}{44\ g\ CO_2} = 1.3\ kg\ C$

c. Determine the length of time it would take to use up all the carbon in 1 square meter of atmosphere.

Solution

Again, this is dimensional analysis:
1.3 kg C $\times \frac{1\ year}{1\ kg\ C} = 1.3$ years.

2.3) Determine the work done for
a. Lifting a 10 kg mass 10 m

Solution

Since we are lifting a weight, we consider gravitational potential:

$$\begin{aligned} w = \int f\ ds &= m \times g \times h \\ &= 10\ kg \times 9.8\ m/s^2 \times 10\ m = 9.8 \times 10^2\ kg \cdot m^2/s^2 \\ &= 9.8 \times 10^2\ J \end{aligned}$$

b. Electrical Work

Solution

$$\begin{aligned} w &= E \times I \times t \\ &= 6.0\ V \times 5.5A\ \times 2\ hours \times \frac{60\ min}{1\ hour} \times \frac{60s}{1\ min} \times \frac{1\ J}{V \cdot A \cdot s} \\ &= 238\ kJ \end{aligned}$$

[1] We mean that 1 mole of atmosphere is 6×10^{23} molecules of atmosphere, randomly chosen. We suppose, here that the atmosphere is approximately all nitrogen and oxygen.

c. Work of a muscle, assuming it acts like a spring

Solution

First, determine the spring constant (k) using the given information:

$$\begin{aligned} force &= k(x - x_0) \\ 5\ N &= k(0.5\ cm) \\ k &= 10\ N/cm \end{aligned}$$

Now, using the spring constant, we can determine the work:

$$\begin{aligned} w &= \frac{k}{2}(x - x_0)^2 \\ &= \frac{10\ N/cm}{2}(11\ cm - 10\ cm)^2 \\ &= \frac{5\ N}{cm} \times 1cm^2 \times \frac{m}{100\ cm} \\ &= 5 \times 10^{-2} J \end{aligned}$$

d. Expansion of a gas against a constant external pressure of 1 atm

Solution

Since the external pressure is constant, than:

$$\begin{aligned} w &= -\int p_{ext}\, dV = -p_{ext} \int dV = -p_{ext} \Delta V \\ &= -1\ atm(3\ L - 1\ L) \\ &= -2\ L \cdot atm \times \frac{101.3\ J}{L \cdot atm} \\ &= -202.6\ J \end{aligned}$$

e. Expansion of a gas against a constant external pressure of 1×10^{-6} atm.

Solution

This is exactly as above, except with changed numbers:

$$\begin{aligned} w &= -p_{ext} \Delta V \\ &= -1 \times 10^{-6}\ atm(3\ L - 1\ L) \\ &= -2 \times 10^{-6}\ L \cdot atm \times \frac{101.3\ J}{L \cdot atm} \\ &= -2.026 \times 10^{-4}\ J \end{aligned}$$

f. Isothermal reversible expansion of a gas.

Solution

First, we will determine the number of moles of gas we have:

$$n = \frac{pV}{RT} = \frac{1\ atm \cdot 1\ L}{(0.08206\ L \cdot atm/K \cdot mol)(298\ K)} = 0.041\ mol$$

Now we can determine what the work must be:

$$w = -\int p_{ext}\, dV = -\int \frac{nRT}{V} dV = -nRT \int \frac{dV}{V}$$

$$
\begin{aligned}
&= -nRT\ln\left(\frac{V_f}{V_i}\right)\\
&= -(0.041\ mol)(8.314\ J/K\cdot mol)(298\ K)\ln\left(\frac{3.0\ L}{1.0\ L}\right)\\
&= -111.6\ J
\end{aligned}
$$

2.8) 1 mole of an ideal monatomic gas (C_v = 3/2 R) at 300 K is expanded from p_i = 10 atm to p_f = 1 atm. Find ΔE, ΔH, q, and w...

a. ... for an isothermal reversible path.

Solution

Since the path is isothermal, than $\Delta T = 0$, so T_f = 300 K.

Since the gas is ideal, the internal energy and enthalpy depend only on the temperature of the gas. Therefore

$\Delta E = \Delta H = 0$

Finally, since the path is reversible, $p_{ext} = p_{int}$, and

$$
\begin{aligned}
w &= -\int p_{ext}\,dV\\
&= -nRT\ \ln\left(\frac{V_f}{V_i}\right)\\
&= -nRT\ \ln\left(\frac{nRT/p_f}{nRT/p_i}\right)\\
&= -nRT\ \ln\left(\frac{p_i}{p_f}\right)\\
&= -(1\ mol)(8.314\ J/K\cdot mol)(300\ K)\ \ln\left(\frac{10\ atm}{1\ atm}\right)\\
&= -5.7\ kJ
\end{aligned}
$$

Finally, since ΔE = q + w = 0, then q = -w = +5.7 kJ.

b. ... for a constant external pressure of 1 atm in an adiabatic system.

Solution

Since the path is adiabatic, q=0. Because ΔE = q + w, then ΔE = w. Since the gas is ideal, then $\Delta E = \frac{3}{2}nR\Delta T$. Setting these equal to one another allows us to solve for the final temperature.

$$
\begin{aligned}
\Delta E = \frac{3}{2}nR\Delta T &= -p_{ext}\Delta V\\
\frac{3}{2}nR(T_f - 300K) &= -1\ atm\left(\frac{nRT_f}{1\ atm} - \frac{nRT_i}{10\ atm}\right)\\
\frac{3}{2}(T_f - 300K) &= -T_f + 30K\\
\frac{5}{2}T_f &= 480K\\
T_f &= 192K
\end{aligned}
$$

Note that in going from the second equation to the third, we have divided each side by nR and multipled out the atm on the right hand side. Using the obtained final temperature, we can determine:

$w = \Delta E = \frac{3}{2}(1mol)(8.314J/K\cdot mol)(192 - 300) = -1.35kJ$

Finally, $\Delta H = \frac{5}{2}nR\Delta T = -2.2kJ$.
c. ... for an adiabatic expansion against a vacuum
Solution
This solution will be filled with Our Very Favorite Number Of All Time[2]
q = 0 (adiabatic path)
$w = -\int p_{ext} dV = 0$ since $p_{ext} = 0$ (definition of a vacuum).
Therefore, $\Delta E = q + w = 0$. If $\Delta E = 0$, $\Delta T = 0$ and $\Delta H = 0$.

2.17) *a.* How much will your temperature raise for every hour you spend gallivanting around the moon in a spacesuit?
Solution

$$\begin{aligned} 4\ kJ/kg \cdot hour &= q = \Delta H = C_p \Delta T \\ 4\ J/g \cdot hour &= 4.18\ J/g \cdot K \times \Delta T \\ 0.957\ K/hour = \Delta T & \end{aligned}$$

So your body temperature increases by 0.957 K every hour.

b. When does it start to become unsafe?
Solution
Your answer here depends upon what you feel would be an unsafe body temperature.

Temperature	Condition	Time to Reach Temperature
37 ° C	normal body temperature	0 hours
38.3 ° C	onset of low-grade fever	1.3 K $\times \frac{1\ hour}{.957\ K}$ = 1.36 hours
40.0 ° C	high-grade fever - possible seizures	3.13 hours
42.2 ° C	risk of brain damage	5.43 hours

2.19) *a.* If the body is 100 % efficient at pushing out gas, how much work is done per day?
Solution

$$\begin{aligned} work\ per\ breath &= -p_{ext}\Delta V \qquad \text{since constant external pressure} \\ &= -(1\ atm)(-0.5\ L) \times \frac{101.3\ J}{L \cdot atm} \\ &= 50.6\ J \end{aligned}$$

Total work per day = 50.6 J / breath × 15000 breaths / day = 760 kJ / day

b. How much mass could you raise 100m to with this amount of work?
Solution

$$\begin{aligned} 760\ kJ = w &= mgh \\ 760 \times 10^3 J \times \frac{kg \cdot m^2/s^2}{1J} &= m(9.8\ m/s^2)(100\ m) \\ 7.6 \times 10^5 kg \cdot m^2/s^2 &= m \times 980\ m^2/s^2 \\ m &= 775\ kg \end{aligned}$$

For reference purposes, a large riding horse has a mass of 500 kg. So you breath around 1.5 horses per day!

[2] Our Very Favorite Number Of All Time is, of course, zero, unless we're dividing by it. This problem contains no division.